FM-40

1) Answer any two questions. $2\times2=4$.

a) If the vectors $3\hat{i}-2\hat{j}+m\hat{k}$ and $-2\hat{i}+\hat{j}+4\hat{k}$ are perpendicular to each other find value of m.

b) Find the angle between the lines

$$\vec{r}=(3\hat{i}+\hat{j}-4\hat{k})+t(\hat{i}+\hat{j}+\hat{k})$$

$$\vec{r}=(5\hat{i}-\hat{j})+8(3\hat{i}+2\hat{j}+4\hat{k})$$

c) Find a vector of magnitude 14 in direction of $-3\hat{i}+6\hat{j}+2\hat{k}$.

2) Answer any three questions. $2\times3=6$.

a) Integrate: $\int\frac{dx}{x^2+x+1}$

b) Find the value of $\int\frac{\sqrt{\sin x}}{\cos^{7/2}x}\,dx$

c) Find $\int_0^2 e^x\,dx$ using definition of definite integral as integral sum. (limit sum)

d) Find differential equation of all circles touching X axis at origin.

e) Solve for y, $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ given $y(0)=0$.

3) Answer both the questions. 5 $\rightarrow 2+3$

a) Find the corner points of LPP,

Max $Z=2x+5y$.

Subject to constraint, 2

$$0\leq x\leq4$$

$$0\leq y\leq3$$

$$x+y\leq6.$$

OR

What is linear programming. Explain with an example.

b) Food F_1 contains 5 units of Vitamin A and 6 units of Vitamin B per gram and cost 20p/gm. F_2 contains 8 units of Vitamin A and 10 units of vitamin B and cost 30p/gm. The daily requirement of A and B is at least 80 and 100 units respectively. Form an LPP to minimize the cost. (3)

OR

Represent graphically,

Minimize $Z = 3x + 2y$

Subject to constraints

$$5x + y \geq 10$$
$$x + y \geq 6$$
$$x + 4y \geq 12$$
$$x \geq 0 \quad y \geq 0$$

4) Answer all the questions. $3 \times 3 = 9$

a) Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at distance $3\sqrt{2}$ units from $(1,2,3)$.

OR

Prove that acute angle between two diagonals of a cube is $\cos^{-1}(\frac{1}{3})$

b) Solve, $\displaystyle\int \frac{\log(1+x^2)}{(x+1)^2}\,dx$

OR

Solve, $\displaystyle\int \frac{dx}{(x-3)\sqrt{2x^2-12x+17}}$

c) Solve for y: $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$ given $y(0)=\frac{1}{2}$

OR

$x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$ given, $y(0)=1$.

5) Solve any two questions. $4\times2=8$.

a) If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$ find the angle between $\vec{a}$ and $\vec{b}$.

b) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$ Prove that, $\vec{a}=\pm2(\vec{b}\times\vec{c})$

c) Find the distance between

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x}{2}=\frac{y-5}{3}=\frac{z+1}{4}.$$

d) Find the distance between,

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$
$$\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(2\hat{i}+3\hat{j}+6\hat{k})$$

6) Answer all the questions. $4\times2=8$.

a) Find $\int_{-\pi/2}^{\pi/2}\frac{dx}{1+\tan^4x}$

OR

Find $\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}}+\frac{1}{\sqrt{2n-2^2}}+\cdots+\frac{1}{n}\right]$

b) Find area in the 1^{st} quadrant which is common to $x^2+y^2=4$ and $x^2+4y^2=9$.

OR

Find the area of $\{(x,y): x^2\leq y\leq |x|\}$