SECTION A

1. C

Sol. The force acting on the particle = $\frac{mdv}{dt}$

Power of the force = $\left(\frac{mdv}{dt}\right)$ v = k (constant)

$$m\frac{v^2}{2} = kt + c \qquad ...(1)$$

at t = 0, v = u $\quad \therefore c = \frac{mu^2}{2}$

Now from (1),

$$m\frac{v^2}{2} = kt + \frac{mu^2}{2}$$

$$\frac{1}{2}m(v^2 - u^2) = kt \qquad ...(2)$$

Again $\frac{mdv}{dt}v = k$

$$m.v\frac{dv}{dx}v = k$$

$$\int_u^v mv^2 dv = \int_0^x k\,dx$$

Intergrating, $\frac{1}{3}m(v^3 - u^3) = kx$ ...(3)

From eqn (2) and (3),

t = $\frac{3}{2}\left(\frac{v^2-u^2}{v^3-u^3}\right)$x

2. A

Sol. From ohm's law

Electric field α current density

3. D

Sol. $e = \int_{-\ell/2}^{\ell/2} B\omega r dr = 0$

4. A

5. B

6. B

Sol. When current is maximum $= \frac{di}{dt} = 0$

7. D

8. A

Sol. Saturation for A and B are different hence intensities are different. Stopping potentials for A and B are equal hence frequencies are equal.

9. B

Sol. Characteristic wavelengths shift of left if z is increased hence B is incorrect

10. D

11. B

Sol. $\rho_1 g h_1^2 = \rho_2 g h_2$

$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$

12. B

13. A

Sol. Process A $\rightarrow$ B is isothermal

Process B $\rightarrow$ C isochoric

Process C $\rightarrow$ A is isobaric

14. C

Sol. The process described is isochoric

Therefore molar heat capacity $= C_v = 2.5R$

15. A

16. B

Sol. Capacitors are in series therefore

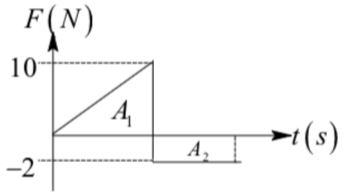
$$\frac{C_1}{C_2}=\frac{V_2}{V_1}=\frac{2}{3}$$

17. B

18. D

19. C

Sol.



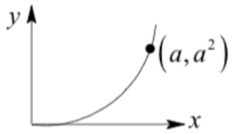
Change in momentum = impulse = Net area under F-t graph

$$=\left(\frac{1}{2}\times 10\times 10\right)-(10\times 2)=30\,kg\,ms^{-1}$$

20. A

Sol. $\overline{F}=xy^2\hat{i}+yx^2\hat{j}$, and displacement

$$d\overline{r}=dx\hat{i}+dy\,\hat{j}$$



$$W=\int_{Path}\overline{F}.d\overline{r}=\int_{Path}\left(xy^2\hat{i}+yx^2\hat{j}\right).\left(dx\hat{i}+dy\,\hat{j}\right)$$

$$=\int_{Path}\left(xy^2dx+yx^2dy\right)=\int_{x=0}^{a}xy^2dx+\int_{y=0}^{y=a^2}yx^2dy$$

$$W=\int_{x=0}^{a}x^5dx+\int_{y=0}^{a^2}y^2dy=\frac{a^6}{6}+\frac{a^6}{3}=\frac{a^6+2a^6}{6}=\frac{a^6}{2}$$

21. 00000.00

Sol. At $t=0, x=\frac{4}{3}\, and\; v=0$

At $x=\frac{4}{3}m, a=3\left(\frac{4}{3}\right)-4=0$

22. 00004.00

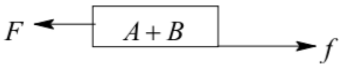
Sol. The distance travelled (here it is displacement) is are under $v-t$ graph

$$=\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 2\times 2=4m$$

23. 00120.00

24. 00003.00

Sol. $F = f = \mu\left(m_A + m_B\right)g = 0.25\times 12 = 3N$



25. 00000.12

26. 00006.00

27. 00001.00

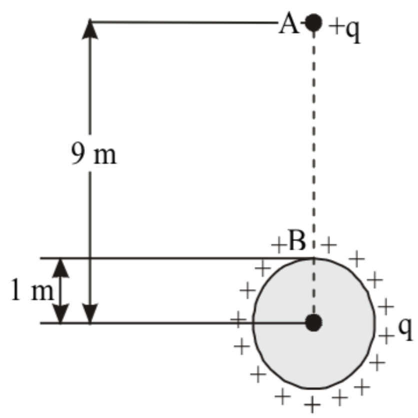
Sol. Use $a = \frac{dV}{ds}$ and find $\frac{dV}{ds}$ using the slope of normal

28. 00060.02

Sol. Since the scale is graduated at 10ºC,
1 cm of the scale at 10ºC = exactly 1 cm
$\therefore$ 1 cm of the scale at 30ºC
= exactly $(1 + 18 \times 10^{-6} \times 20)$cm
$\therefore$ 60 cm of the scale at 30ºC
= exactly $60.00\,(1 + 36 \times 10^{-5})$
= 60.02 cm

29. 00028.00

Sol. Keeping in mind that here both electric and gravitational potential energy are changing and for external point a charged sphere behaves as whole of its charge were concentrated at its centre, applying conservation of energy between initial and final position, we have



$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9} + mg \times 9 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{1} + mg \times 1$$

or $q^2 = \frac{80\times 10^{-3}\times 9.8}{10^9}$

or $q = 28\ \mu C$

30. 06000.00

Sol. The fringe–width β is given by

$$\beta = \frac{\lambda D}{2d} \quad \text{and} \quad \beta' = \frac{\lambda D'}{2d}$$

where λ is the wavelength of light used, D is the distance of the screen from the two slits and 2 d is the separation between two slits.

Now $\beta - \beta' = \frac{\lambda(D - D')}{2d}$

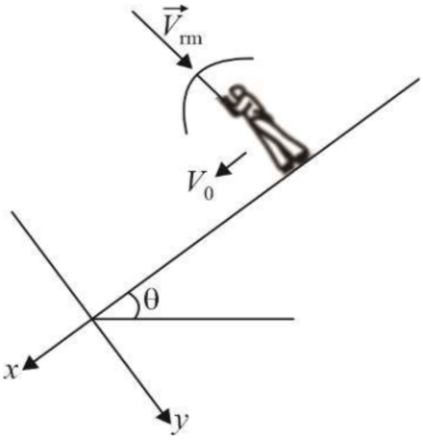
$\Rightarrow \lambda = \frac{(\beta - \beta')2d}{D - D'}$

$\lambda = \frac{(3\times 10^{-5})(10^{-3})}{5\times 10^{-2}}$

$= 0.6 \times 10^{-6}$ m = 6000 Å

SECTION-B

1.(B) Take x axis along the incline and y direction perpendicular to it.



Velocity of rain relative to the man is perpendicular to the incline in this case (i.e., along the umbrella stick. This keeps canopy perpendicular to the rainfall and provides maximum safety).

$$\vec{V}_{rm} = \vec{V}_r - \vec{V}m$$

$$= (V_x\hat{i} + V_y\hat{j}) - V_0\hat{i} = (V_x - V_0)\hat{i} + V_y\hat{j}$$

Since $\vec{V}_{rm}$ has no x component

$$\therefore \quad V_x = V_0$$

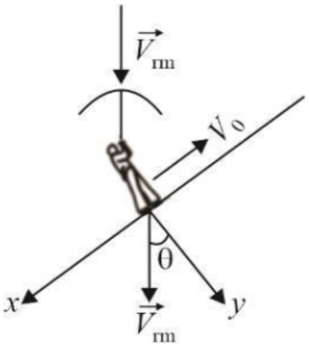
When the man is walking up, $\vec{V}_{rm}$ is directed vertically downward.

$$\vec{V}_{rm} = (V_x\hat{i} + V_y\hat{j}) - (-V_0\hat{i})$$

$$= V_0\hat{i} + V_y\hat{j} + V_0\hat{i}$$

$$= 2V_0\hat{i} + V_y\hat{j}$$

From diagram



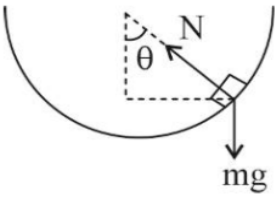
$$\tan\theta = \frac{2V_0}{V_y}$$

$$\frac{3}{4} = \frac{2V_0}{V_y} \Rightarrow V_y = \frac{8V_0}{3} \qquad \therefore \qquad V_r = \sqrt{V_x^2 + V_y^2} = \frac{\sqrt{73}}{3}V_0$$

2.(B) $\cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60^\circ$

$$N\cos\theta = mg$$

$$N\sin\theta = mw^2(R\sin\theta)$$



$$\tan\theta = \frac{w^2 R\sin\theta}{g}$$

$$\sqrt{3} = \frac{w^2 R}{g}\frac{\sqrt{3}}{2} \Rightarrow w = \sqrt{\frac{2g}{R}}$$

3.(D) The elevator moves up with constant acceleration, hence y-t graph must be a parabla.

Let $y=kt^2$

At $t=2, y=4$

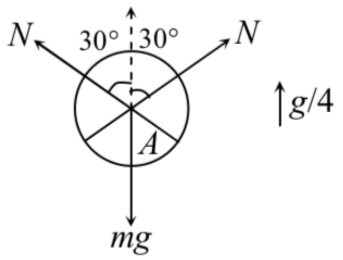
$\therefore \quad k=1 \quad \therefore \quad y=t^2 \quad \therefore \quad \frac{dy}{dt}=2t=4m/s \quad (at\ t=2)$

$\frac{d^2y}{dt^2}=2.0\,m/s^2$

In the reference frame of the elevator the acceleration of bolt is $12m/s^2$ and its initial velocity is zero. Time required for a displacement of $1.5m$ in this frame is $y=\frac{1}{2}\times 12\times t^2$

$1.5=\frac{1}{2}\times 12\times t^2 \Rightarrow t=0.5s \quad \therefore$ Bolt hits the floor at $t = 2.5\ s$

4.(B) For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$



5.(A) $\overrightarrow{v_B}=8\hat{j}+2t\hat{k}$

$\overrightarrow{v_c}=\overrightarrow{v_0}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$

$\overrightarrow{r_B}=8t\hat{j}+t^2\hat{k}$

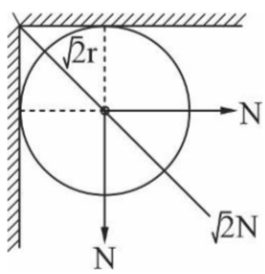
$\overrightarrow{r_c}=v_xt\hat{i}+v_yt\hat{j}+v_zt\hat{k}$

At 4 sec, $\overrightarrow{r_B}=\overrightarrow{r_C} \Rightarrow v_x=0, v_y=8\,m/s$ and $v_z=4m/s \quad \therefore \quad \overrightarrow{v_0}=(8\hat{j}+4\hat{k})\,m/s$

SECTION–C

6.(B) $a=-s$, $\quad v\frac{dv}{ds}=-s$, $\quad \int_{v_0}^{0} v\,dv=-\int_{0}^{s} s\,ds$, $\quad \frac{v_0^2}{2}=\frac{s^2}{2} \Rightarrow s=v_0$

7.(AC) The thread makes an angle of $\sin^{-1}\left(\frac{\sqrt{2}r}{2r}\right)=45°$ with the vertical.



If N normal force by each wall on the cylinder

$\sqrt{2}N=\frac{T}{\sqrt{2}}$ and $Mg=\frac{T}{\sqrt{2}}$

$\Rightarrow \quad T=\sqrt{2}Mg$ and $N=\frac{Mg}{\sqrt{2}}$

8.(AB) For completing circle $u = \sqrt{4gl}$

Speed of bob when rod becomes horizontal can be found by energy conservation

$$\frac{1}{2}m(4gl) = \frac{1}{2}mv^2 + mgl \Rightarrow v = \sqrt{2gl}$$

Radial acceleration, $a_r = \frac{v^2}{l} = 2g$

Tangential acceleration, $a_t = \frac{mg}{m} = g$ $\therefore$ Net acceleration $= \sqrt{a_r^2 + a_t^2} = \sqrt{5}g$

At the topmost point, velocity is zero

So, force by rod on bob is mg radially outwards. So, C and D are incorrect

9.(BD) If the height of the pan at an instant is *l* and the boy transfers a small mass *m* of the sand into it, he performs a work = mg *l* (against gravity).

This work done increases the gravitational potential energy of the sand mass which loses a part of it in compressing the spring.

Therefore, total work done by the boy will be finally found as gravitational potential energy of the sand plus the elastic potential energy of the spring.

As per the question.

$$K\frac{L_0}{2} = Mg \qquad \ldots.(i)$$

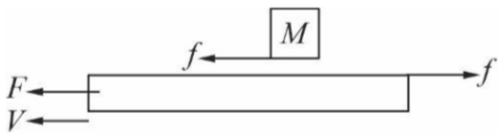
Work done $W = Mg\frac{L_0}{2} + \frac{1}{2}K\left(\frac{L_0}{2}\right)^2$

$= Mg\frac{L_0}{2} + \frac{1}{8}KL_0^2 = Mg\frac{L_0}{2} + \frac{1}{4}MgL_0$ [using (1)]

$= \frac{3}{4}MgL_0$

10.(ABC) Friction force bwtween the block and the plank $= f$

Acceleration of the block $a = \frac{f}{M}$.



If the slipping stops in time 't'

$V = 0 + at$

$$V = \frac{ft}{M} \qquad \ldots.(1)$$

For the plank external force F = friction (f)

Power of the external force P = FV

Work done by external force in time 't'

$W = Pt = FVt = ftV = MV^2$ [$\because$ $ft = MV$ from (1)]

The work done by the external force can be interpreted as

W = Heat produced + gain in kE of the block

$\therefore$ Heat $= MV^2 - \frac{1}{2}MV^2 = \frac{1}{2}MV^2$

$\frac{1}{2}MV^2$ amount of KE. This energy has been lost as heat.